

Quantum Monte Carlo calculations of the equation of state of neutron matter with chiral EFT interactions

Ingo Tews

(Institute for Nuclear Theory Seattle)

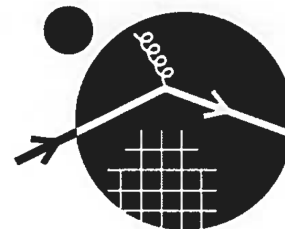
In collaboration with

A. Gezerlis, J. Carlson, S. Gandolfi, J. Lynn, A. Schwenk,
E. Kolomeitsev, J. Lattimer, A. Ohnishi

Extracting Bulk Properties of Neutron-Rich Matter with Transport Models in Bayesian Perspective,
April 4th, 2017, FRIB-MSU, East Lansing



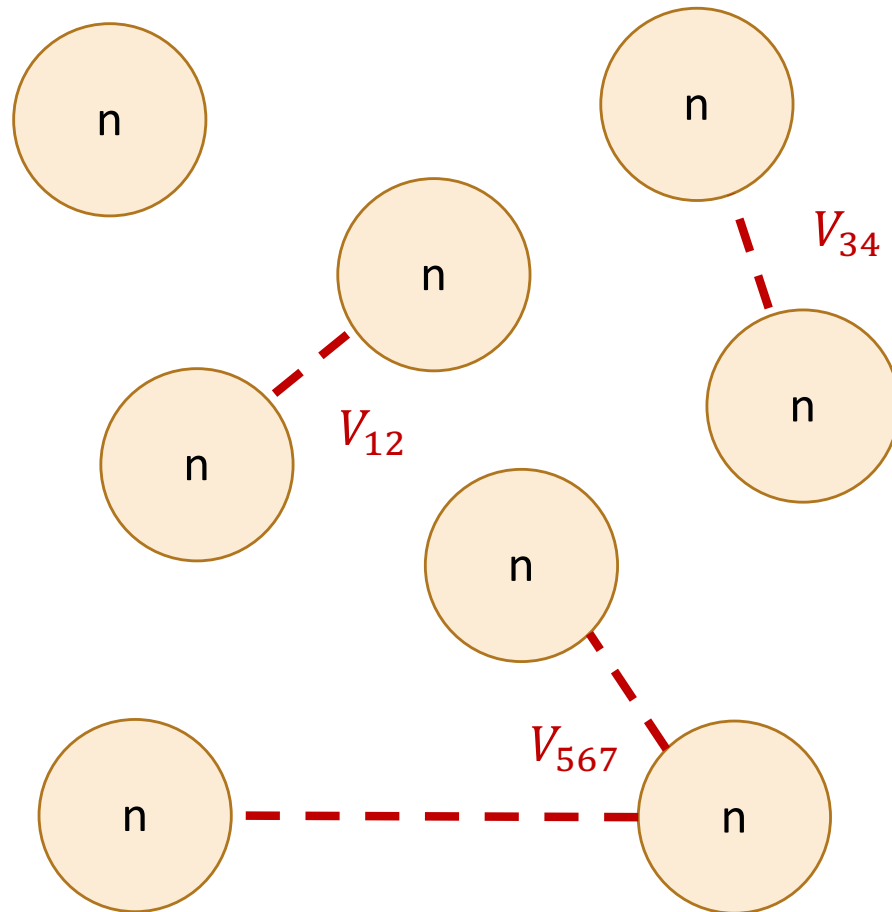
JINA-CEE



INSTITUTE for
NUCLEAR THEORY

- Motivation
- Chiral effective field theory *e.g. Epelbaum *et al.*, PPNP (2006) and RMP (2009)*
 - **Systematic basis** for nuclear forces, naturally includes **many-body forces**
 - **Very successful** in calculations of nuclei and nuclear matter
- Quantum Monte Carlo method
 - **Very precise** for strongly interacting systems
 - Need of **local interactions** (depend only on $r = |\mathbf{r}_i - \mathbf{r}_j|$)
- **Local chiral interactions** *Gezerlis, IT, et al., PRL & PRC (2013, 2014, 2016)*
 - Can be constructed **up to N²LO**
- Results for neutron matter, light nuclei, and n-alpha scattering
- S and L constraints from lower bound of neutron matter

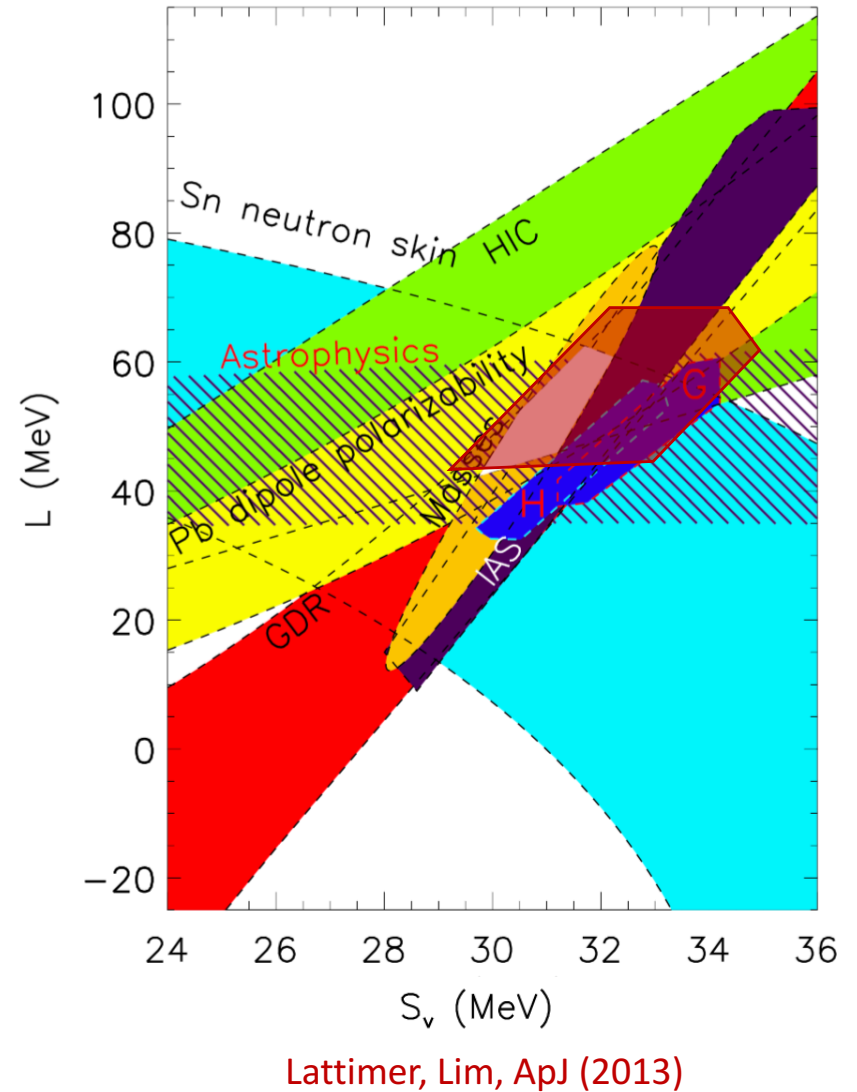
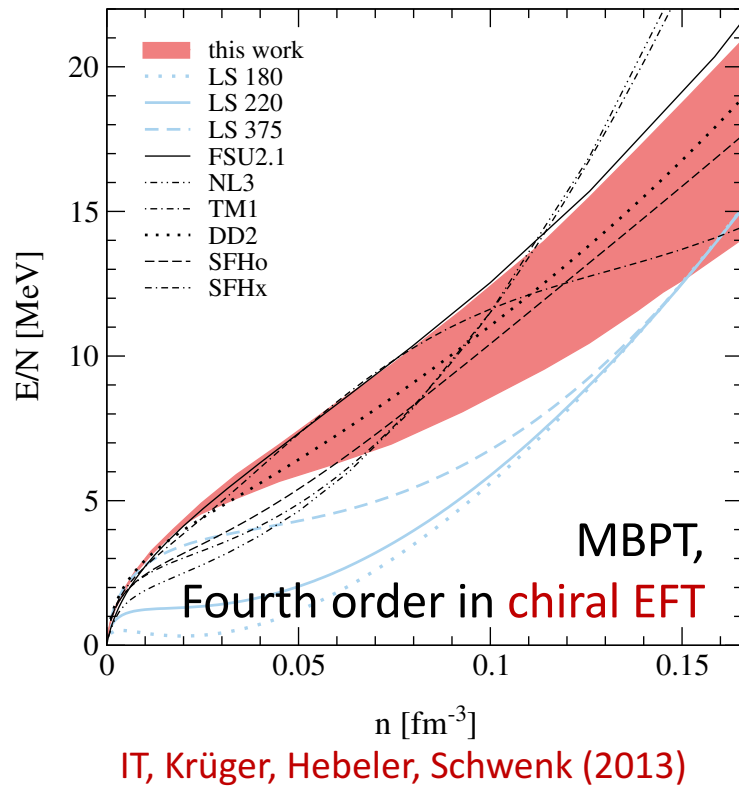
Motivation



To obtain the equation of state we need:

- ❑ A theory for the strong interactions among nucleons
 - Phenomenological forces or **Chiral EFT**
- ❑ An ab initio method to solve the many-body Schrödinger equation
 - Many-body Pert. Theory (MBPT), **Quantum Monte Carlo (QMC)**, Coupled Cluster, ...

Motivation

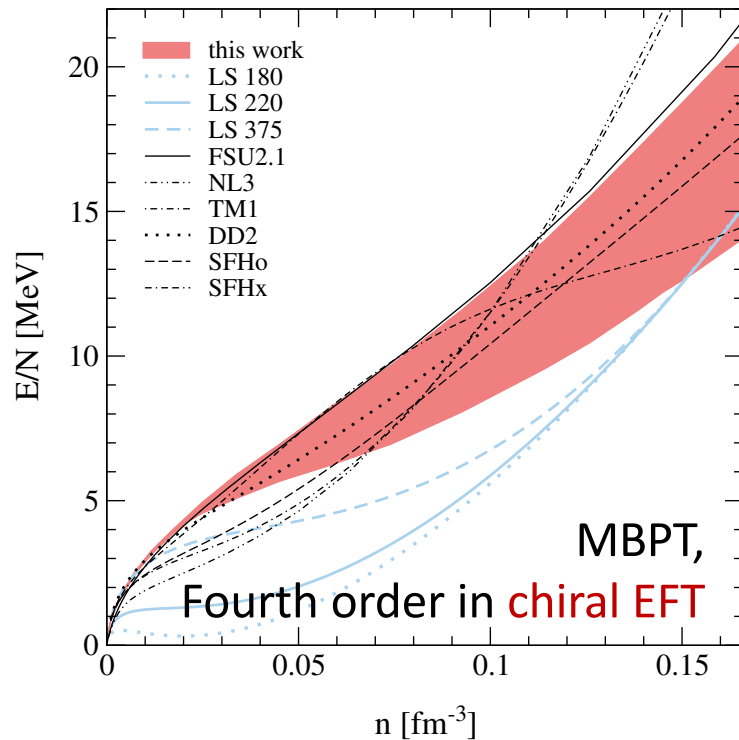


Put constraints on **symmetry energy** and its density dependence L :

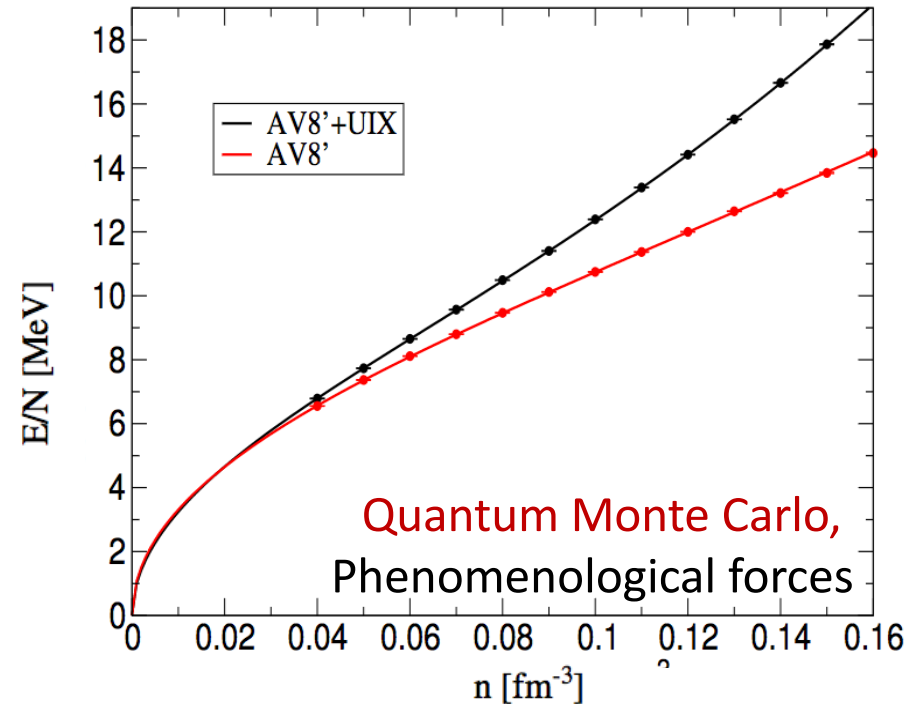
- $S_V = 28.9 - 34.9$ MeV
- $L = 43.0 - 66.6$ MeV

Good agreement with experimental constraints

Motivation



IT, Krüger, Hebeler, Schwenk (2013)



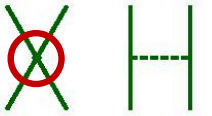
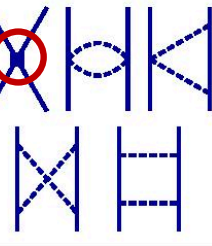

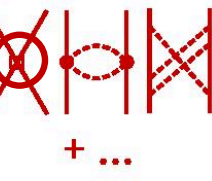
Credit: Stefano Gandolfi

Status:

- Sizeable uncertainty for chiral EFT calculations of neutron matter
- Quantum Monte Carlo: **very precise** method for strongly interacting systems
- Phenomenological interactions provide a good description of light nuclei and nuclear matter, but it is **not clear how to systematically improve their quality**, no uncertainty estimates

➤ QMC calculations with local chiral EFT interactions

Chiral effective field theory for nuclear forces

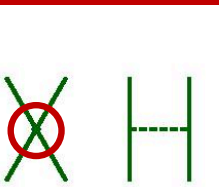


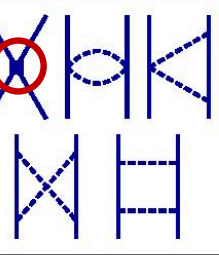
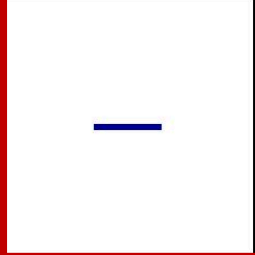
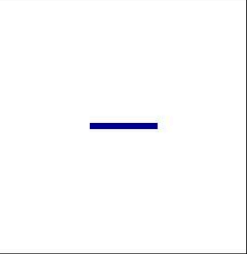
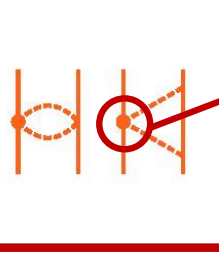
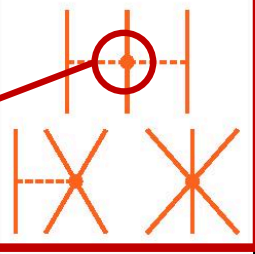
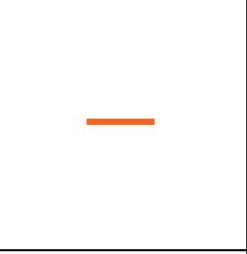
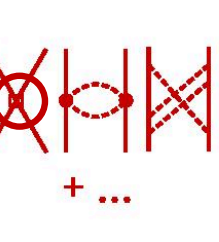
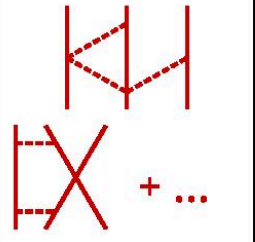
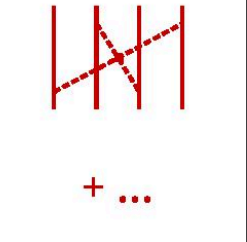
		NN		
LO	$\mathcal{O}\left(\frac{Q^0}{\Lambda^0}\right)$		2 LECs	
NLO	$\mathcal{O}\left(\frac{Q^2}{\Lambda^2}\right)$		7 LECs	
N ² LO	$\mathcal{O}\left(\frac{Q^3}{\Lambda^3}\right)$			
N ³ LO	$\mathcal{O}\left(\frac{Q^4}{\Lambda^4}\right)$	 + ...	15 LECs	

Systematic expansion of nuclear forces in low momenta Q over breakdown scale Λ_b :

- Pions and nucleons as explicit degrees of freedom
- Long-range physics **explicit**, short-range physics expanded in **general operator basis**, couplings (LECs) fit to experiment
- Separation of scales:
Expand in powers of $\left(\frac{Q}{\Lambda_b}\right)^{\nu} \sim \left(\frac{1}{3}\right)^{\nu}$
- Power counting scheme
- Can work to desired accuracy with **systematic error estimates**

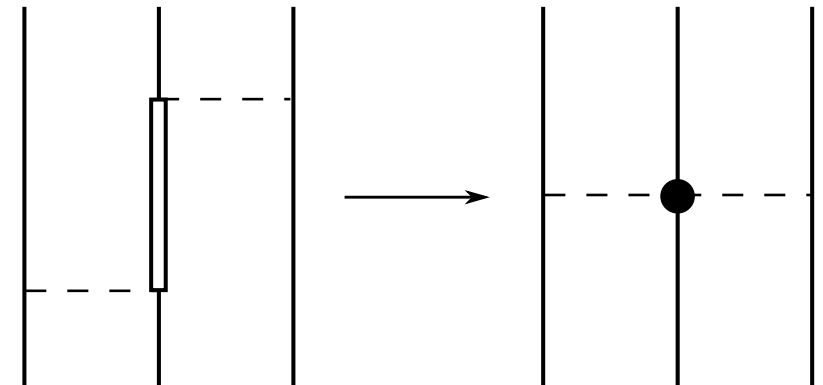
Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Kaiser, Machleidt, Meißner, Hammer ...

Chiral effective field theory for nuclear forces

		NN	3N	4N
LO	$\mathcal{O}\left(\frac{Q^0}{\Lambda^0}\right)$			
NLO	$\mathcal{O}\left(\frac{Q^2}{\Lambda^2}\right)$			
N ² LO	$\mathcal{O}\left(\frac{Q^3}{\Lambda^3}\right)$			
N ³ LO	$\mathcal{O}\left(\frac{Q^4}{\Lambda^4}\right)$			

Many-body forces:

- Crucial for nuclear physics
- **Natural hierarchy of nuclear forces**
- **Fitting:** NN forces in NN system (NN phase shifts), 3N forces in 3N/4N system (Binding energies, radii)
- **Consistent interactions:** Same couplings for two-nucleon and many-body sector



Quantum Monte Carlo method

Solve the many-body Schrödinger equation

$$H |\psi\rangle = -\frac{\partial}{\partial \tau} |\psi\rangle, \quad \tau = it$$
$$\psi(R, \tau) = \int dR'^{3N} \langle R | e^{-(T+V)\tau} | R' \rangle \psi(R', 0)$$

Basic steps:

- Choose **trial wavefunction** which overlaps with the ground state

$$|\psi(R, 0)\rangle = |\psi_T(R, 0)\rangle = \sum_i c_i |\phi_i\rangle \rightarrow \sum_i c_i e^{-(E_i - E_0)\tau} |\phi_i\rangle$$

- **Evaluate propagator** for small timestep $\Delta\tau$, feasible **only for local potentials**
- Make **consecutive small time steps** using Monte Carlo techniques to project out ground state

$$|\psi(R, \tau)\rangle \rightarrow |\phi_0\rangle \quad \text{for} \quad \tau \rightarrow \infty$$

More details:

Carlson, Gandolfi, Pederiva, Pieper, Schiavilla, Schmidt, Wiringa, RMP (2015)

Quantum Monte Carlo method

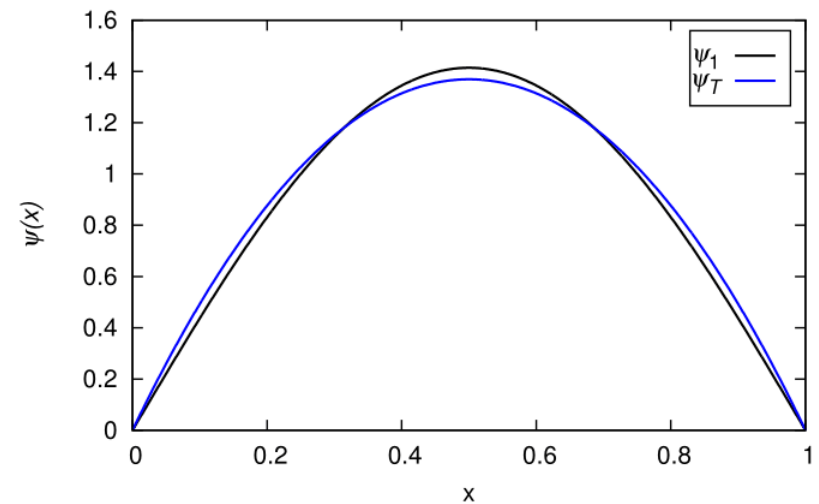
Particle in a 1D box, solution:

$$\psi_n(x) = \sqrt{2} \sin(n\pi x), \quad E_n = \frac{n^2 \pi^2}{2}$$

Basic steps:

- Choose parabolic **trial wavefunction** which overlaps with the ground state

Animation by Joel Lynn, TU Darmstadt



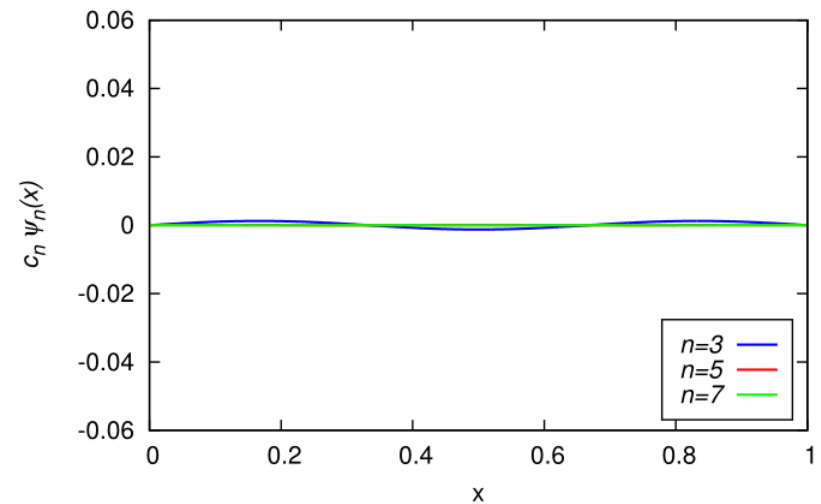
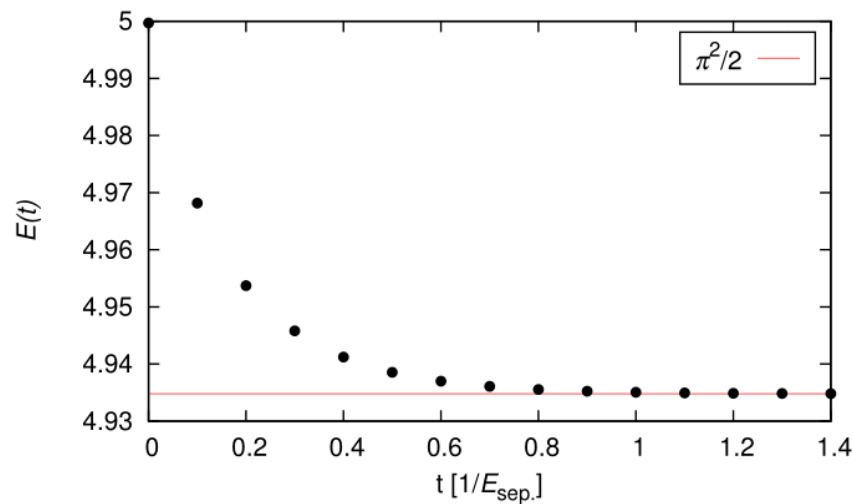
Quantum Monte Carlo method

Particle in a 1D box, solution:

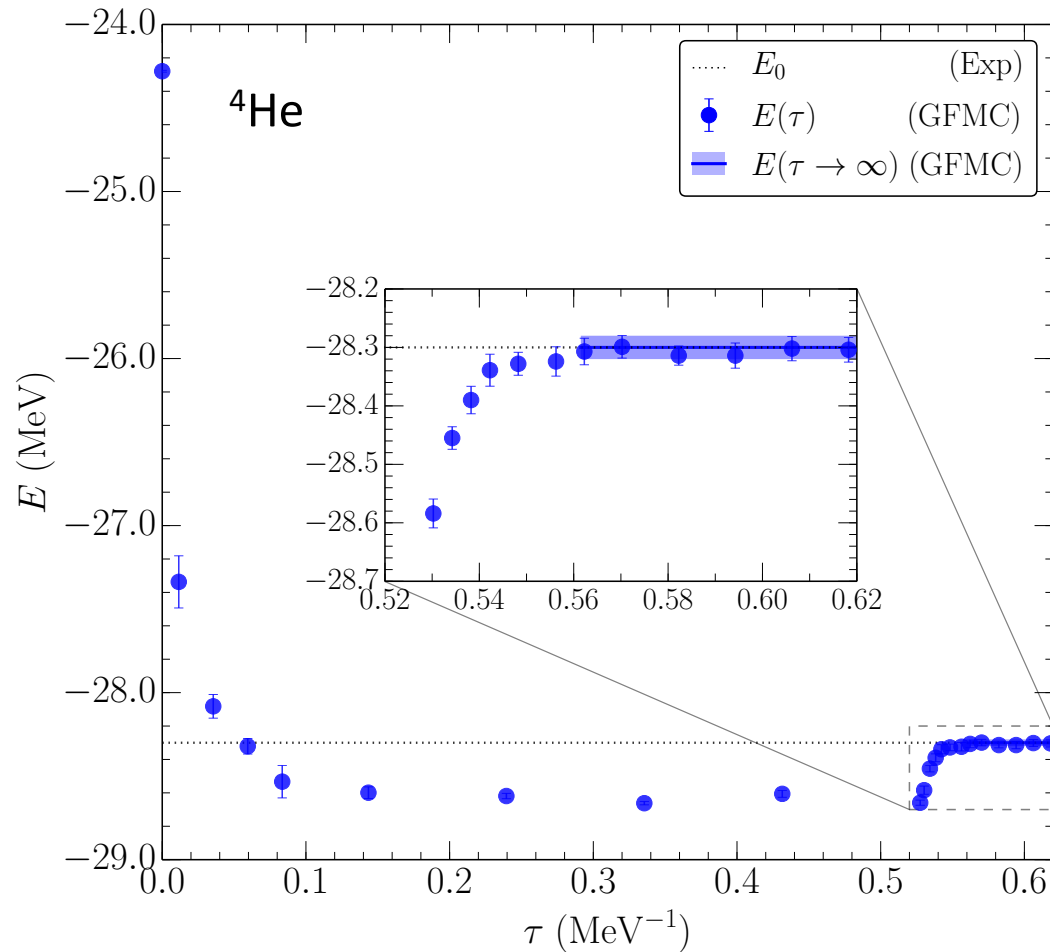
$$\psi_n(x) = \sqrt{2} \sin(n\pi x), \quad E_n = \frac{n^2 \pi^2}{2}$$

➤ Make **consecutive small timesteps**, $\tau = 1.4 \left(\frac{1}{E_{\text{sep}}} \right)$

Animation by Joel Lynn, TU Darmstadt

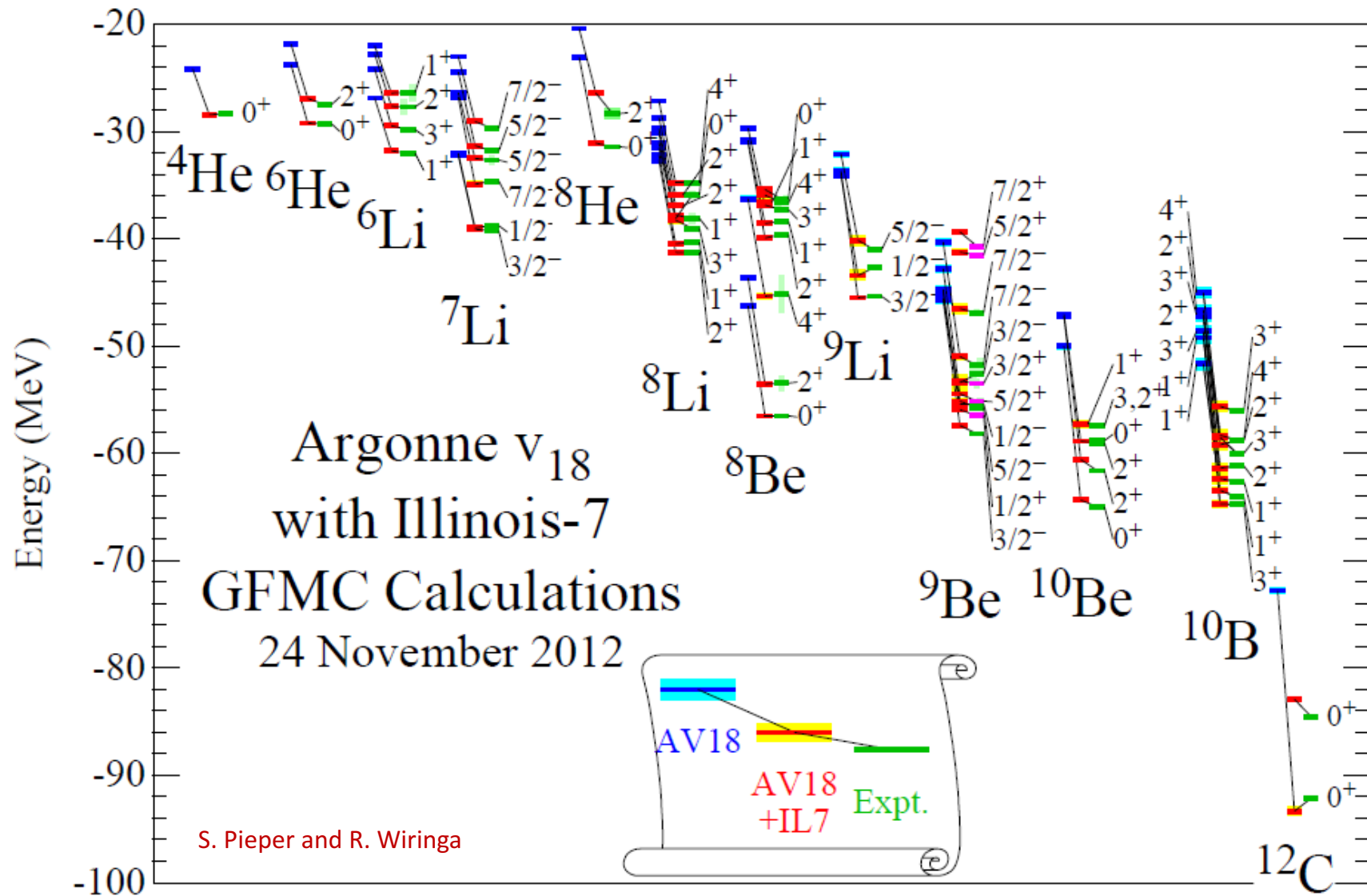


Quantum Monte Carlo method



Lynn, IT, Carlson, Gandolfi, Gezerlis, Schmidt, Schwenk, in preparation.

Quantum Monte Carlo method



To evaluate the propagator for small timesteps $\Delta\tau$ we need **local potentials**:

$$\langle r' | \hat{V} | r \rangle = \begin{cases} V(r) \delta(r - r'), & \text{if local} \\ V(r', r), & \text{if nonlocal} \end{cases}$$

Chiral Effective Field Theory interactions generally nonlocal:

- Momentum transfer $\mathbf{q} \rightarrow \mathbf{p}' - \mathbf{p}$
- Momentum transfer in the exchange channel $\mathbf{k} = \frac{1}{2}(\mathbf{p} + \mathbf{p}')$
- **Fourier transformation**: $\mathbf{q} \rightarrow \mathbf{r}, \mathbf{k} \rightarrow \text{Derivatives}$

Sources of nonlocalities:

- Usual **regulator** in relative momenta

$$f(\mathbf{p}) = e^{-(\mathbf{p}/\Lambda)^{2n}}$$

- k-dependent **contact operators**

Solutions:


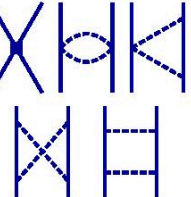
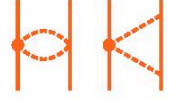
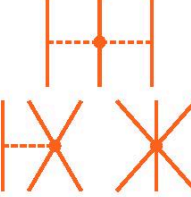

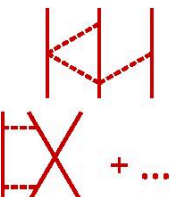

- Choose **local regulators**:

$$V_{\text{long}}(r) \rightarrow V_{\text{long}}(r) \left(1 - e^{-(r/R_0)^4}\right)$$

$$\delta_{R_0}(\mathbf{r}) = \alpha e^{-(r/R_0)^4}$$

- Use Fierz freedom to choose **local set of contact operators**

Local chiral interactions

		NN	3N	4N
LO	$O\left(\frac{Q^0}{\Lambda^0}\right)$		—	—
NLO	$O\left(\frac{Q^2}{\Lambda^2}\right)$		—	—
N ² LO	$O\left(\frac{Q^3}{\Lambda^3}\right)$			—
N ³ LO	$O\left(\frac{Q^4}{\Lambda^4}\right)$			

- Leading order $V^{(0)} = V_{\text{cont}}^{(0)} + V^{\text{OPE}}$
- Pion exchange local → **local regulator**

$$f_{\text{long}}(r) = 1 - \exp(-r^4/R_0^4)$$

- Contact potential:

$$V_{\text{cont}}^{(0)} = \alpha_1 \mathbf{1} + \alpha_2 \sigma_1 \cdot \sigma_2 + \alpha_3 \tau_1 \cdot \tau_2 + \alpha_4 \sigma_1 \cdot \sigma_2 \tau_1 \cdot \tau_2$$


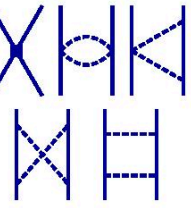

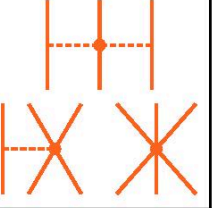
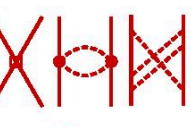
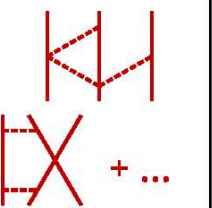

→ Only two independent (Pauli principle)

$$V_{\text{cont}}^{(0)} = C_S \mathbf{1} + C_T \sigma_1 \cdot \sigma_2$$

$$f_{\text{short}}(r) = \alpha \exp(-r^4/R_0^4)$$

Weinberg, van Kolck, Kaplan, Savage, Wise,
Epelbaum, Kaiser, Machleidt, Meißner, Hammer ...

Local chiral interactions




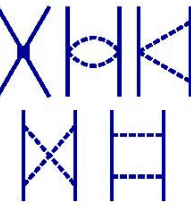


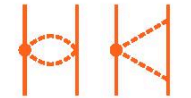
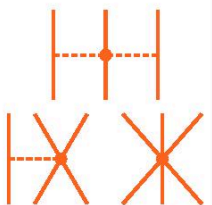

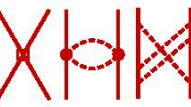
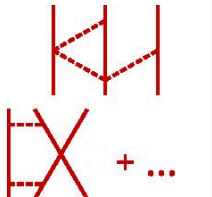

		NN	3N	4N
LO	$O\left(\frac{Q^0}{\Lambda^0}\right)$		—	—
NLO	$O\left(\frac{Q^2}{\Lambda^2}\right)$		—	—
N ² LO	$O\left(\frac{Q^3}{\Lambda^3}\right)$			—
N ³ LO	$O\left(\frac{Q^4}{\Lambda^4}\right)$			

➤ Choose local set of short-range operators at NLO (7 out of 14)

$$\begin{aligned}
 V_{\text{cont}}^{(2)} = & \gamma_1 q^2 + \gamma_2 q^2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + \gamma_3 q^2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \\
 & + \gamma_4 q^2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \\
 & + \gamma_5 k^2 + \gamma_6 k^2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + \gamma_7 k^2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \\
 & + \gamma_8 k^2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \\
 & + \gamma_9 (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)(\mathbf{q} \times \mathbf{k}) \\
 & + \gamma_{10} (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)(\mathbf{q} \times \mathbf{k}) \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \\
 & + \gamma_{11} (\boldsymbol{\sigma}_1 \cdot \mathbf{q})(\boldsymbol{\sigma}_2 \cdot \mathbf{q}) \\
 & + \gamma_{12} (\boldsymbol{\sigma}_1 \cdot \mathbf{q})(\boldsymbol{\sigma}_2 \cdot \mathbf{q}) \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \\
 & + \gamma_{13} (\boldsymbol{\sigma}_1 \cdot \mathbf{k})(\boldsymbol{\sigma}_2 \cdot \mathbf{k}) \\
 & + \gamma_{14} (\boldsymbol{\sigma}_1 \cdot \mathbf{k})(\boldsymbol{\sigma}_2 \cdot \mathbf{k}) \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 .
 \end{aligned}$$

Weinberg, van Kolck, Kaplan, Savage, Wise,
Epelbaum, Kaiser, Machleidt, Meißner, Hammer ...

Local chiral interactions

		NN	3N	4N
LO	$O\left(\frac{Q^0}{\Lambda^0}\right)$			
NLO	$O\left(\frac{Q^2}{\Lambda^2}\right)$			
N ² LO	$O\left(\frac{Q^3}{\Lambda^3}\right)$			
N ³ LO	$O\left(\frac{Q^4}{\Lambda^4}\right)$	 + ...	 + ...	 + ...

Weinberg, van Kolck, Kaplan, Savage, Wise,
Epelbaum, Kaiser, Machleidt, Meißner, Hammer ...

➤ Choose local set of short-range operators at NLO (7 out of 14)

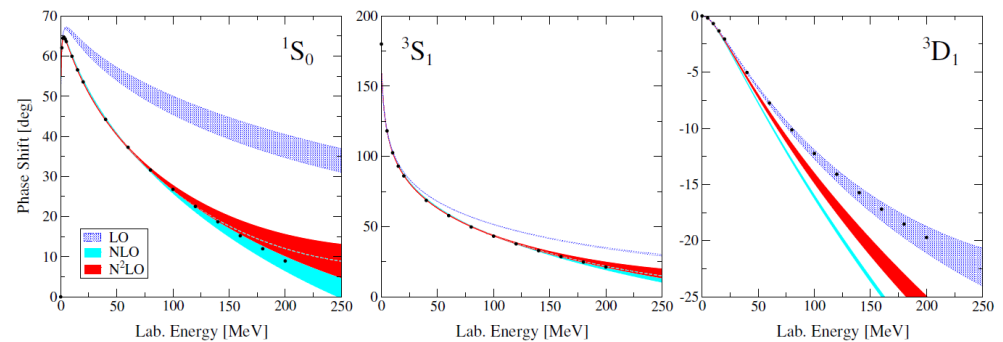
➤ Pion exchanges up to N²LO are local

➤ This freedom can be used to remove all nonlocal operators up to N²LO

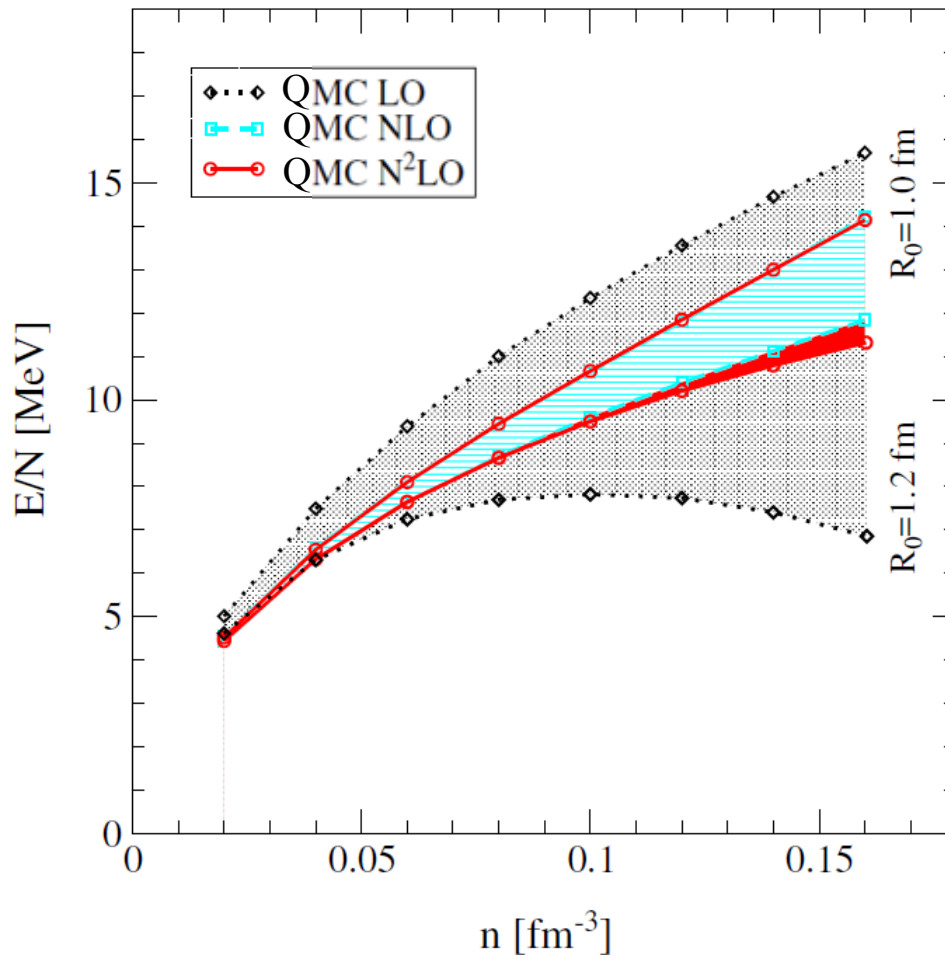
Gezerlis, IT, Epelbaum, Gandolfi, Hebeler, Nogga, Schwenk, PRL (2013)

Gezerlis, IT, Epelbaum, Freunek, Gandolfi, Hebeler, Nogga, Schwenk, PRC (2014)

➤ LECs fit to phase shifts



QMC results for NN forces

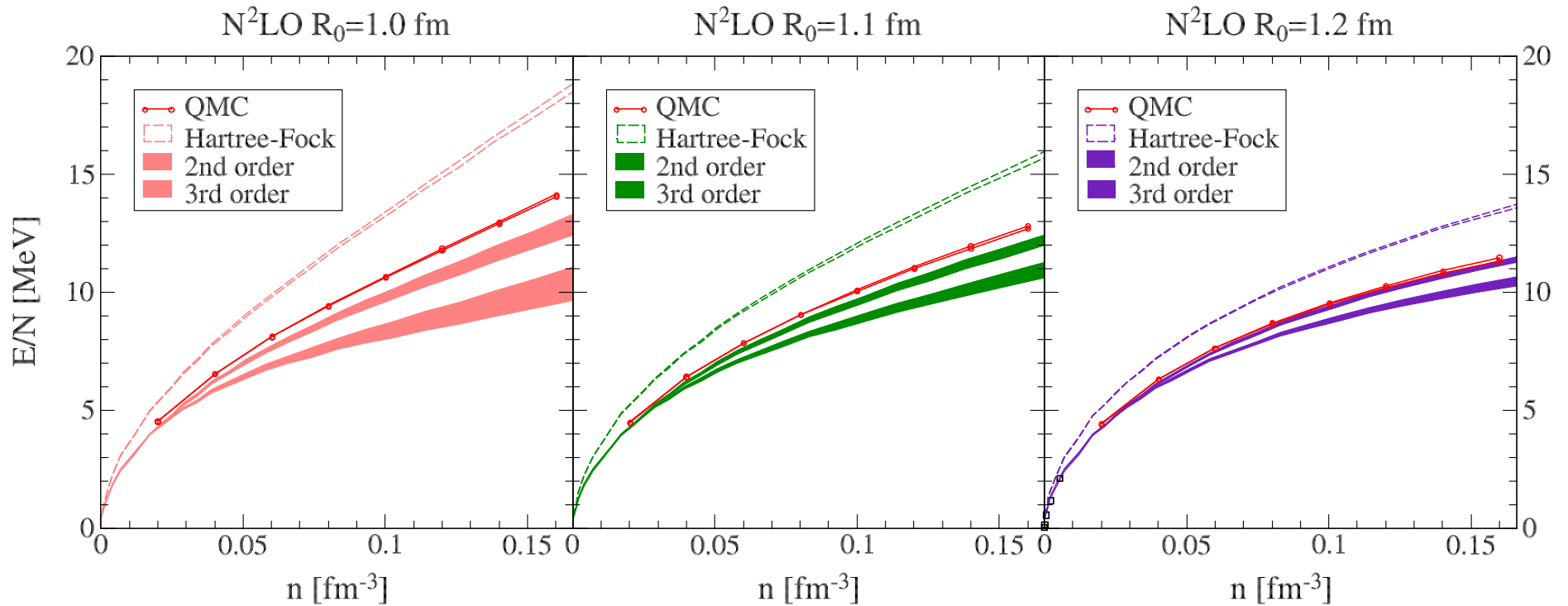


Gezerlis, IT, Epelbaum, Freunek, Gandolfi, Hebeler, Nogga,
Schwenk, PRL (2013) and PRC (2014)

NN-only calculation:

- QMC:
Statistical uncertainty of points negligible
- Bands include NN cutoff variation
 $R_0 = 1.0 - 1.2$ fm
- Order-by-order convergence up to saturation density

Benchmark of MBPT




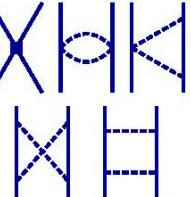


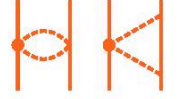
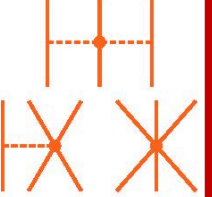


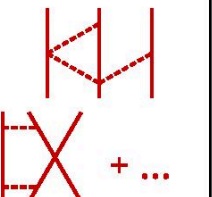



Gezerlis, IT, Epelbaum, Freunek, Gandolfi, Hebeler, Nogga, Schwenk, PRC (2014)

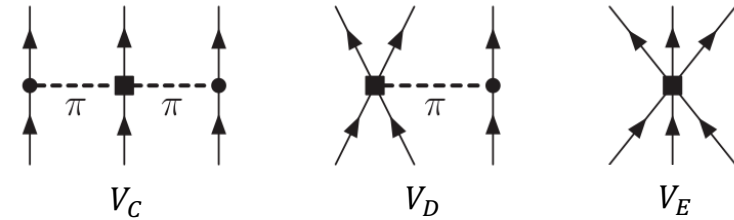
Many-body perturbation theory:

- Excellent agreement with QMC for soft potentials ($R_0 = 1.2$ fm)
- **Validates perturbative calculations** for those interactions

Local chiral interactions

		NN	3N	4N
LO	$\mathcal{O}\left(\frac{Q^0}{\Lambda^0}\right)$			
NLO	$\mathcal{O}\left(\frac{Q^2}{\Lambda^2}\right)$			
N ² LO	$\mathcal{O}\left(\frac{Q^3}{\Lambda^3}\right)$			
N ³ LO	$\mathcal{O}\left(\frac{Q^4}{\Lambda^4}\right)$	 + ...	 + ...	 + ...

Inclusion of **leading 3N forces**:



Three topologies:

- Two-pion exchange V_C
- One-pion-exchange contact V_D
- Three-nucleon contact V_E

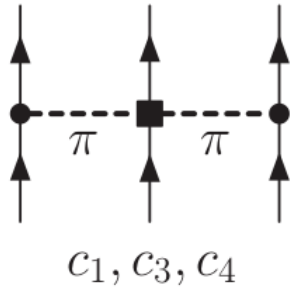
Only two new couplings: c_D and c_E .

Fit to uncorrelated observables:

- Probe properties of light nuclei: ${}^4\text{He } E_B$
- Probe T=3/2 physics: n- α scattering

Weinberg, van Kolck, Kaplan, Savage, Wise,
Epelbaum, Kaiser, Machleidt, Meißner, Hammer ...

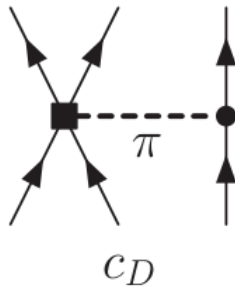
QMC with chiral 3N forces



Usually Two-pion-exchange most important in PNM:

c_1 term: Tucson-Melbourn S-wave interaction

$c_{3,4}$ term: Fujita-Miyazawa interaction

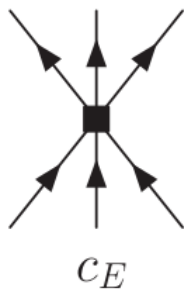


Usually V_D and V_E vanish in neutron matter:

c_D due to spin-isospin structure,

c_E due to Pauli principle

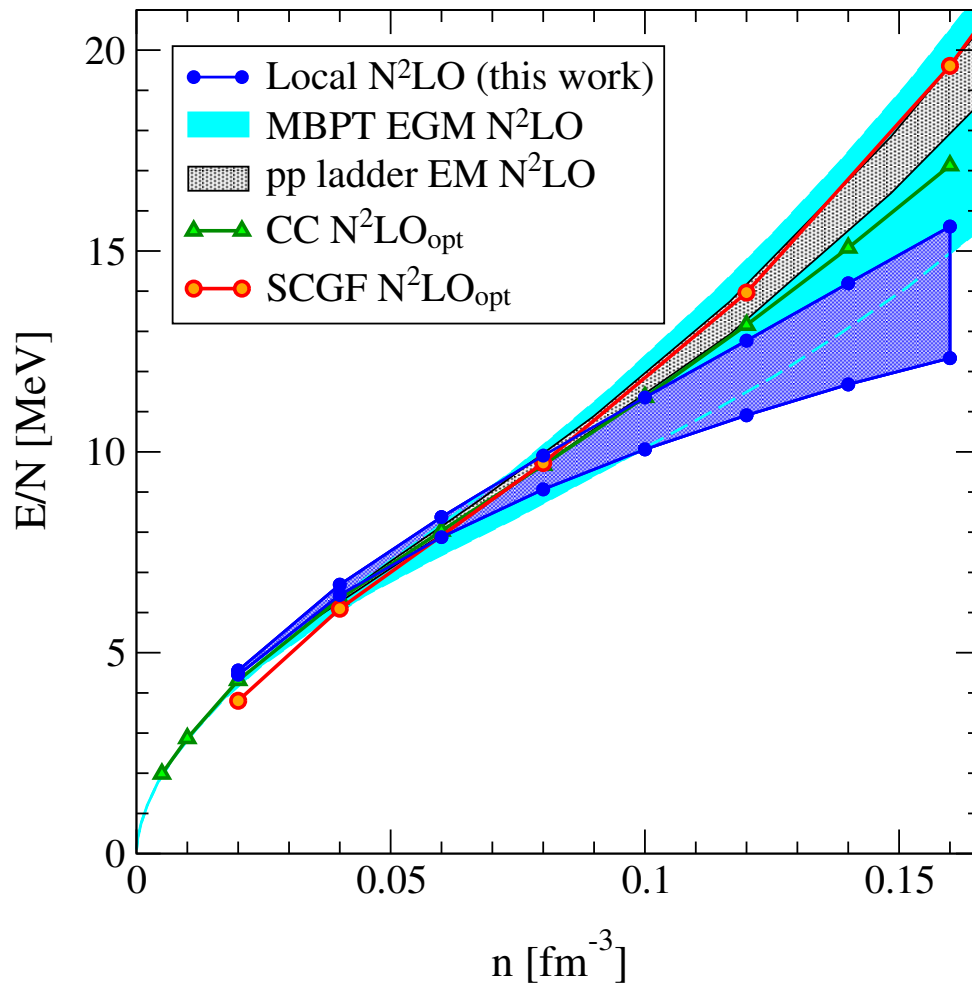
see also Hebeler, Schwenk, PRC (2010)



Only true for regulator symmetric in particle labels like

commonly used nonlocal regulators, **not for local regulators**

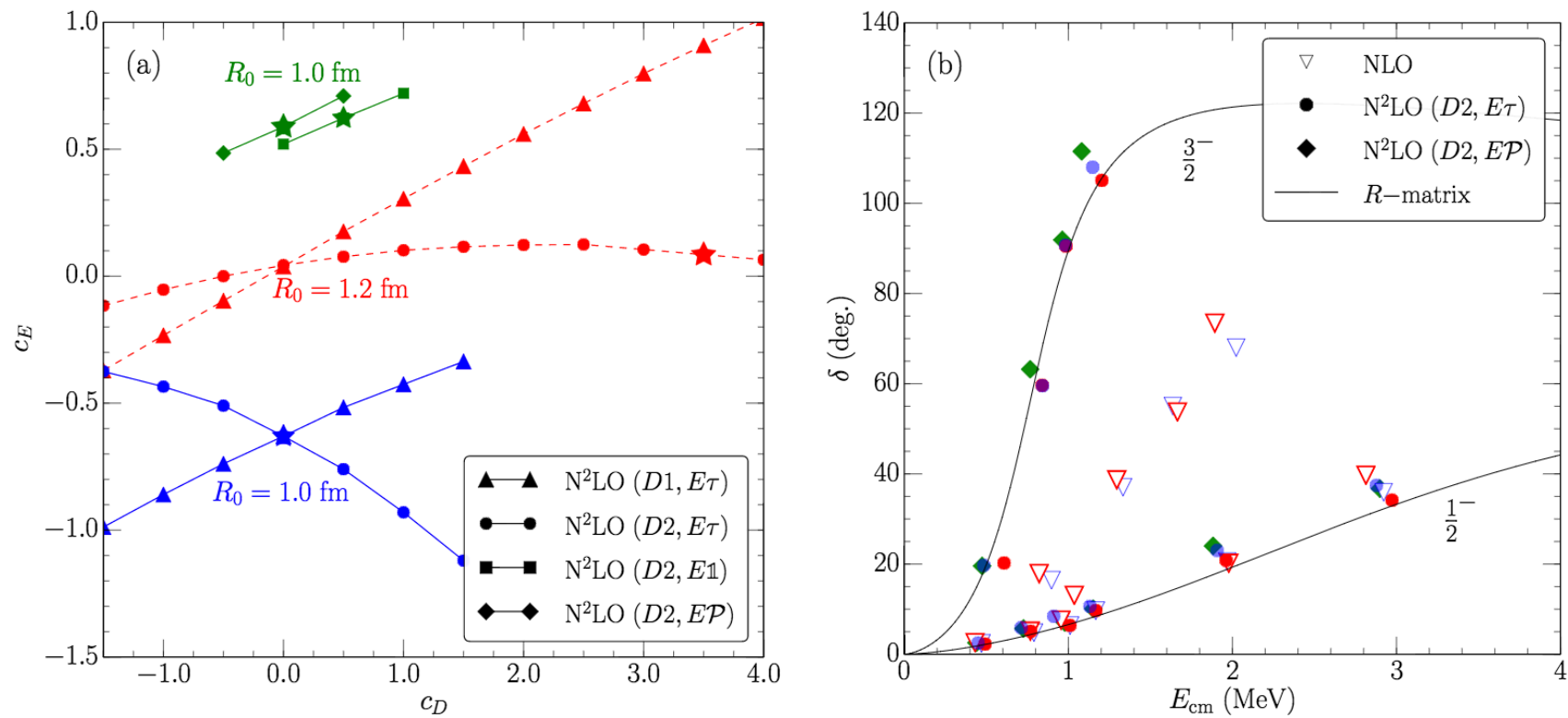
QMC results with 3N TPE



IT, Gandolfi, Gezerlis, Schwenk, PRC (2016)

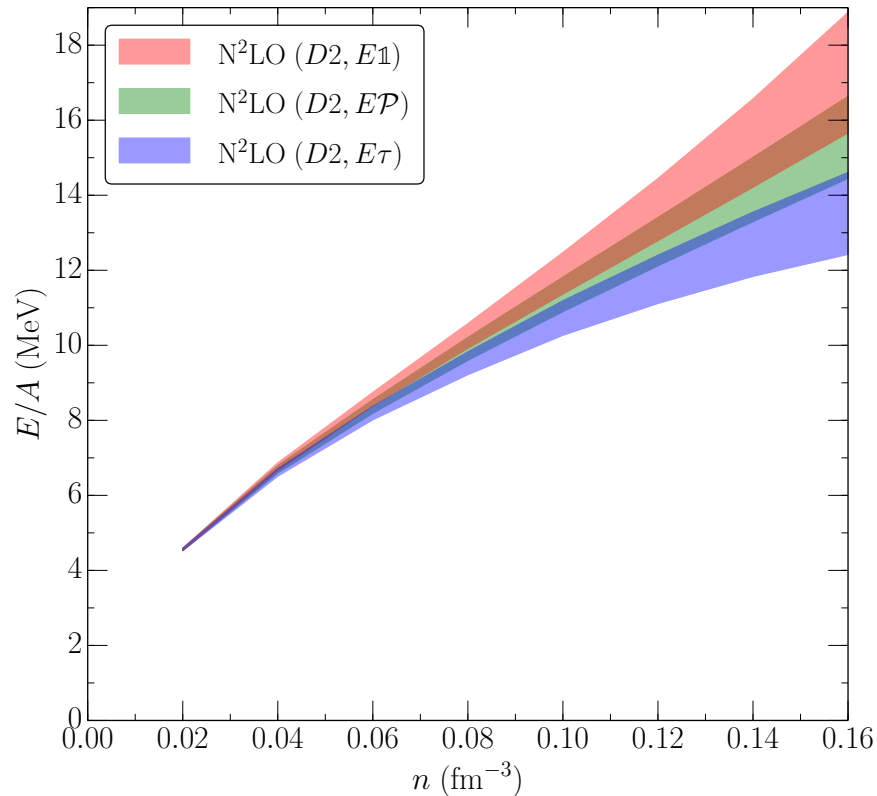
- Only three-nucleon **two-pion exchange**
 $\sim c_1$ and c_3
- Auxiliary-field diffusion Monte Carlo:
 - NN + **3N TPE forces**
 - $R_0 = 1.0 - 1.2$ fm
 - $R_{3N} = 1.0 - 1.2$ fm
- 3N cutoff dependence small
- TPE 3N contributions $\approx 1 - 2$ MeV at n_0
- smaller than for nonlocal regulators

➤ Fit c_E and c_D to ${}^4\text{He}$ binding energy and n - α scattering

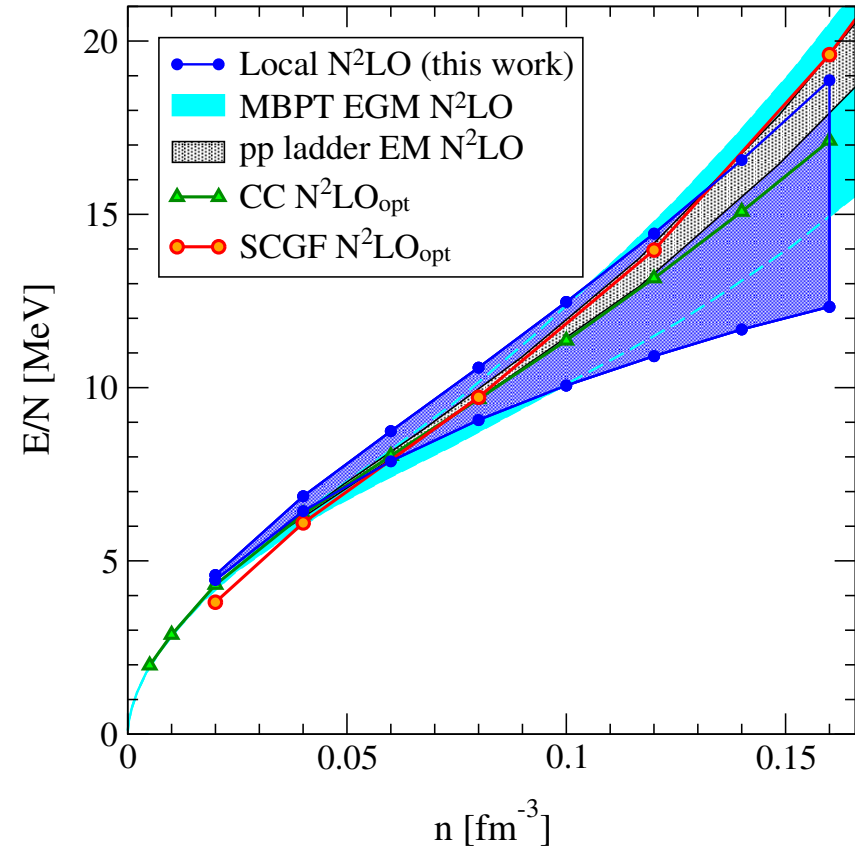


Lynn, IT, Carlson, Gandolfi, Gezerlis, Schmidt, Schwenk, PRL (2016)

Results

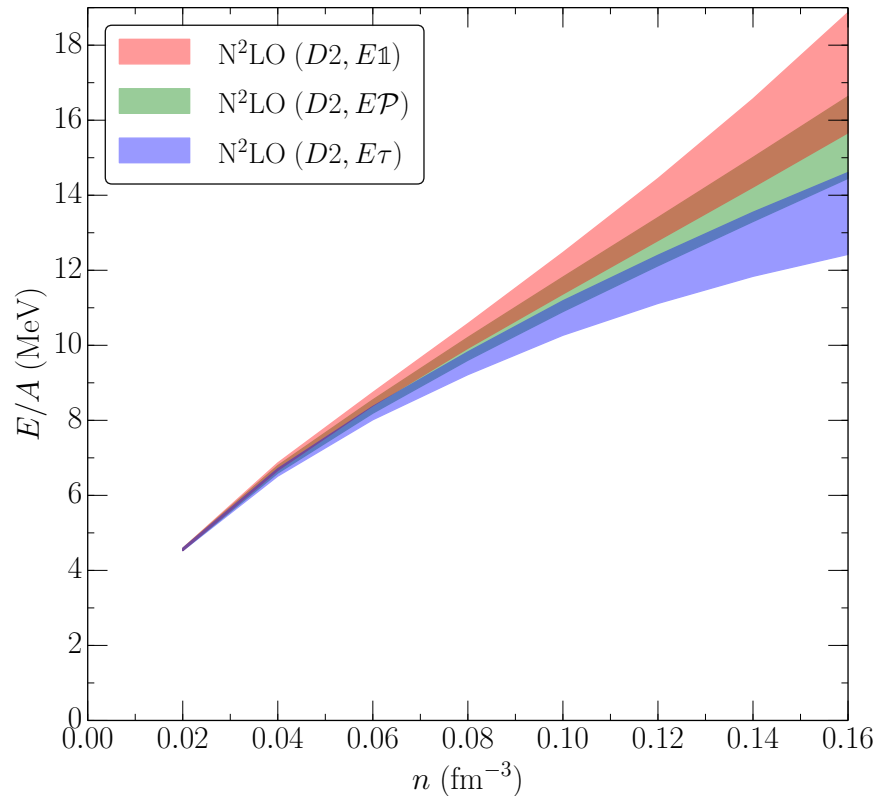


Lynn, IT, Carlson, Gandolfi, Gezerlis, Schmidt, Schwenk,
PRL (2016)

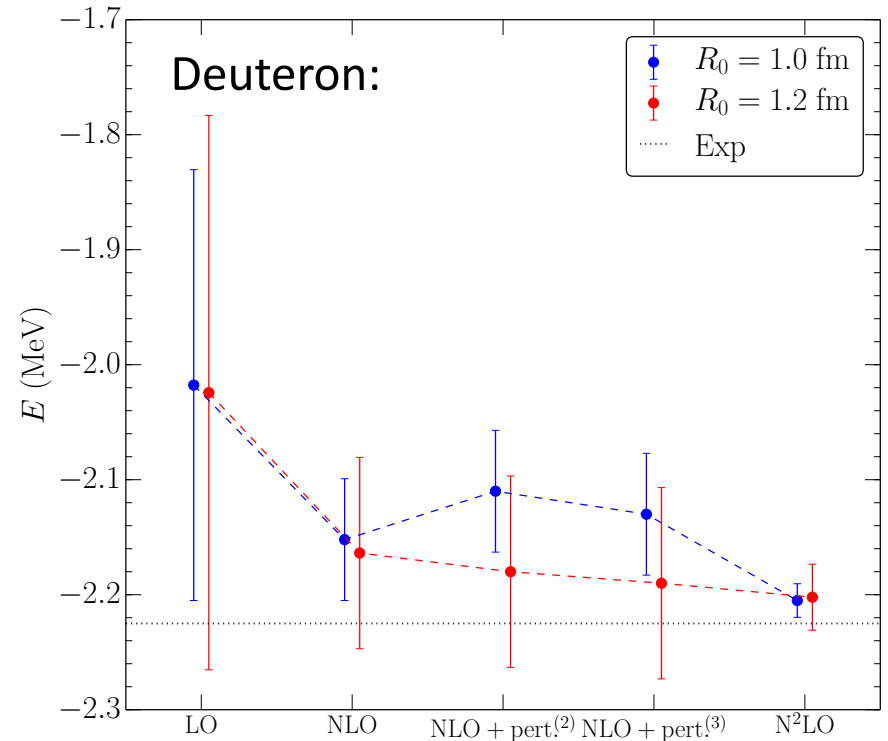


- Less repulsion from TPE, but additional contributions due to shorter-range 3N forces
- After inclusion of all contributions we find agreement of various approaches
(different way of uncertainty estimate, see EKM, PRC 2015)

Results



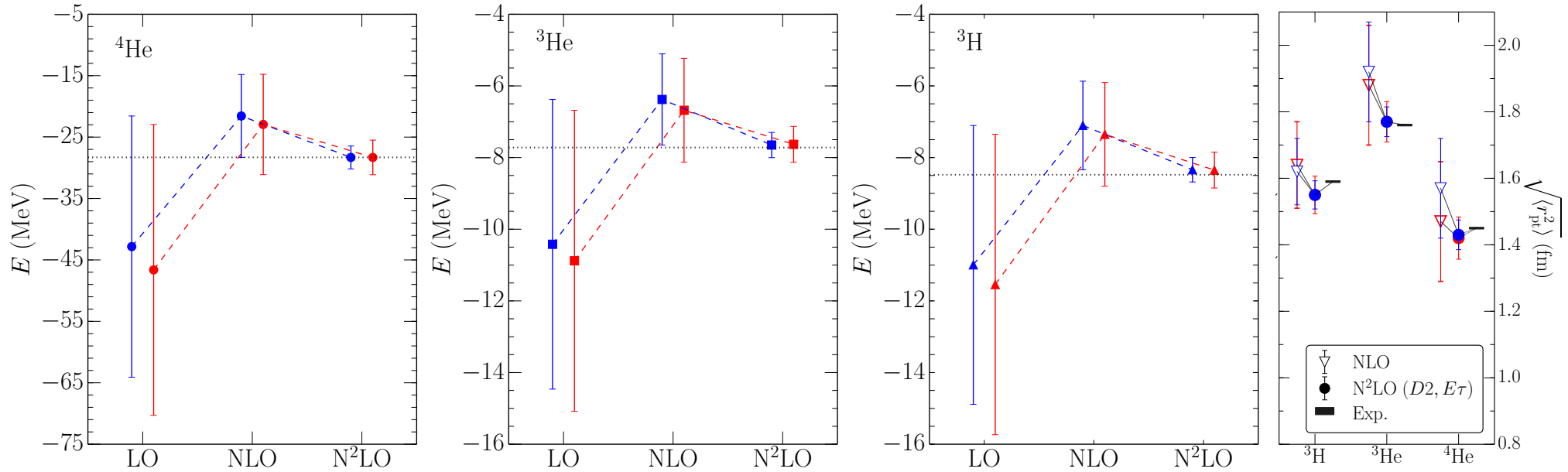
Lynn, IT, Carlson, Gandolfi, Gezerlis, Schmidt, Schwenk, PRL (2016)



Lynn, IT, Carlson, Gandolfi, Gezerlis, Schmidt, Schwenk, in preparation.

- Chiral interactions at N²LO simultaneously reproduce the properties of $A \leq 5$ systems and of neutron matter (uncertainty estimate as in E. Epelbaum et al, EPJ (2015))
- Commonly used phenomenological 3N interactions fail for neutron matter
Sarsa, Fantoni, Schmidt, Pederiva, PRC (2003)

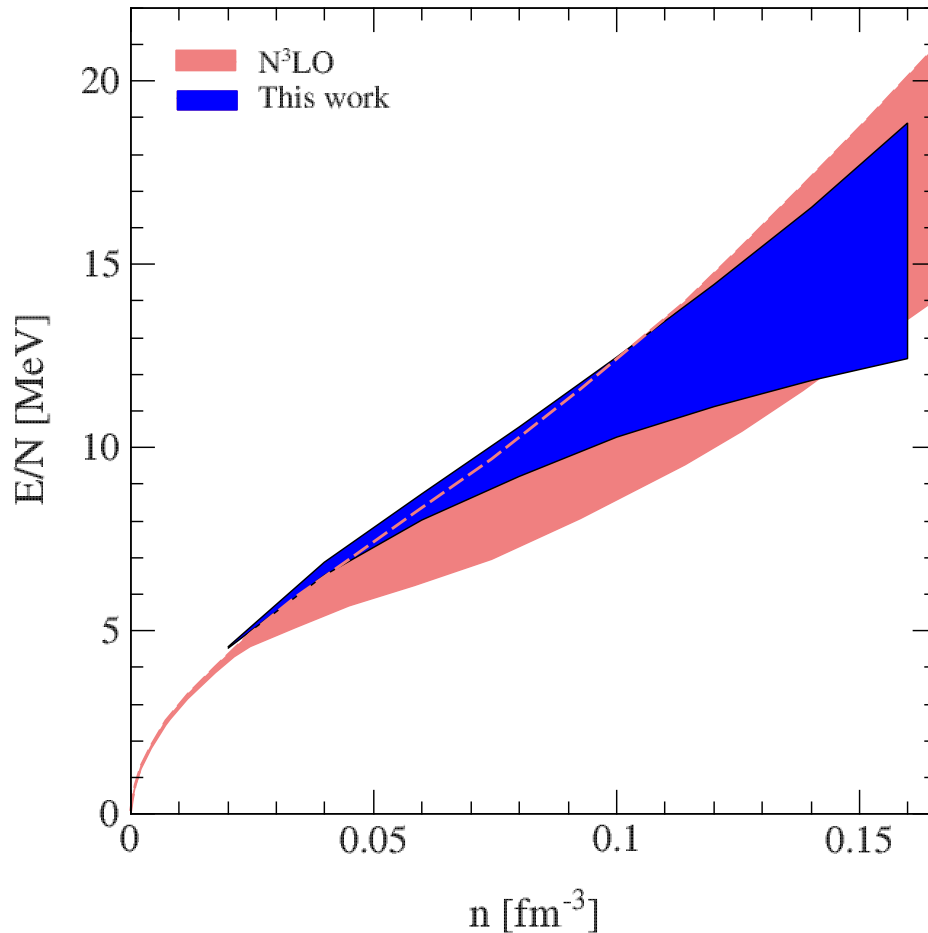
Results



Lynn, IT, Carlson, Gandolfi, Gezerlis, Schmidt, Schwenk, in preparation.

- Chiral interactions at N^2LO simultaneously reproduce the properties of $A \leq 5$ systems and of neutron matter (uncertainty estimate as in E. Epelbaum et al, EPJ (2015))
- Commonly used phenomenological 3N interactions fail for neutron matter
Sarsa, Fantoni, Schmidt, Pederiva, PRC (2003)

Comparing to N³LO calculation:



IT, Krüger, Hebeler, Schwenk, PRL (2013)

Lynn, IT, Carlson, Gandolfi, Gezerlis, Schmidt, Schwenk, PRL (2016)

Chiral EFT forces with the
Quantum Monte Carlo method:

- Energies agree well with MBPT result within uncertainty bands
- Many-body uncertainty negligible
- **uncertainties comparable** but QMC band only at N²LO and includes also hard interactions

- Improve local chiral interactions:
 - Develop N³LO potentials

Next step: N³LO

Improve local chiral interactions:

- Develop maximally local N³LO potentials
- Inclusion of Delta degree of freedom

➤ Problem: only **8 out of 30** possible operators local

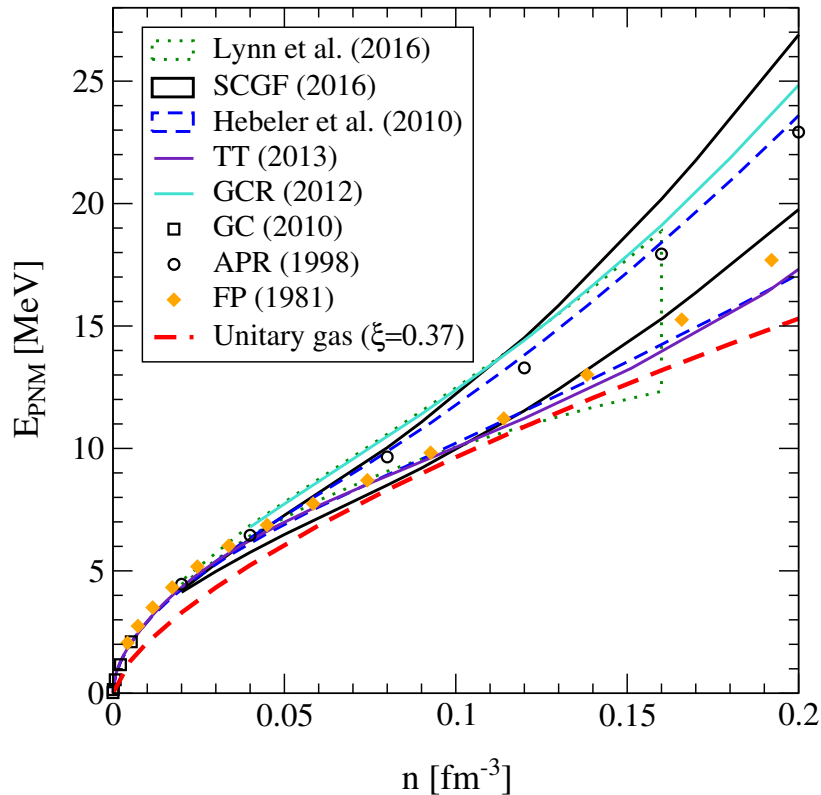
$$\begin{aligned}
 V_{\text{cont}}^{(4)} = & D_1 \mathbf{q}^4 + D_2 \mathbf{q}^4 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + D_3 \mathbf{q}^4 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + D_4 \mathbf{q}^4 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \\
 & + D_5 \mathbf{k}^4 + D_6 \mathbf{k}^4 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + D_7 \mathbf{k}^4 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + D_8 \mathbf{k}^4 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \\
 & + D_9 \mathbf{q}^2 \mathbf{k}^2 + D_{10} \mathbf{q}^2 \mathbf{k}^2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + D_{11} \mathbf{q}^2 \mathbf{k}^2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + D_{12} \mathbf{q}^2 \mathbf{k}^2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \\
 & + D_{13} (\mathbf{q} \times \mathbf{k})^2 + D_{14} (\mathbf{q} \times \mathbf{k})^2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + D_{15} (\mathbf{q} \times \mathbf{k})^2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + D_{16} (\mathbf{q} \times \mathbf{k})^2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \\
 & + \frac{i}{2} D_{17} \mathbf{q}^2 (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot (\mathbf{q} \times \mathbf{k}) + \frac{i}{2} D_{18} \mathbf{q}^2 (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot (\mathbf{q} \times \mathbf{k}) \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \\
 & + \frac{i}{2} D_{19} \mathbf{k}^2 (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot (\mathbf{q} \times \mathbf{k}) + \frac{i}{2} D_{20} \mathbf{k}^2 (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot (\mathbf{q} \times \mathbf{k}) \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \\
 & + D_{21} \mathbf{q}^2 \boldsymbol{\sigma}_1 \cdot \mathbf{q} \boldsymbol{\sigma}_2 \cdot \mathbf{q} + D_{22} \mathbf{q}^2 \boldsymbol{\sigma}_1 \cdot \mathbf{q} \boldsymbol{\sigma}_2 \cdot \mathbf{q} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \\
 & + D_{23} \mathbf{k}^2 \boldsymbol{\sigma}_1 \cdot \mathbf{q} \boldsymbol{\sigma}_2 \cdot \mathbf{q} + D_{24} \mathbf{k}^2 \boldsymbol{\sigma}_1 \cdot \mathbf{q} \boldsymbol{\sigma}_2 \cdot \mathbf{q} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \\
 & + D_{25} \mathbf{q}^2 \boldsymbol{\sigma}_1 \cdot \mathbf{k} \boldsymbol{\sigma}_2 \cdot \mathbf{k} + D_{26} \mathbf{q}^2 \boldsymbol{\sigma}_1 \cdot \mathbf{k} \boldsymbol{\sigma}_2 \cdot \mathbf{k} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \\
 & + D_{27} \mathbf{k}^2 \boldsymbol{\sigma}_1 \cdot \mathbf{k} \boldsymbol{\sigma}_2 \cdot \mathbf{k} + D_{28} \mathbf{k}^2 \boldsymbol{\sigma}_1 \cdot \mathbf{k} \boldsymbol{\sigma}_2 \cdot \mathbf{k} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \\
 & + D_{29} ((\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot (\mathbf{q} \times \mathbf{k}))^2 + D_{30} ((\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot (\mathbf{q} \times \mathbf{k}))^2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2
 \end{aligned} \tag{34}$$

➤ But: work in progress!

Now: Constraints on S and L from
lower bound of neutron matter energy

Kolomeitsev, Lattimer, Ohnishi, IT, [arXiv:1611.07133](https://arxiv.org/abs/1611.07133)

S and L constraints from lower bound of neutron matter energy



Kolomeitsev, Lattimer, Ohnishi, IT, arXiv:1611.07133

Empirical observation:

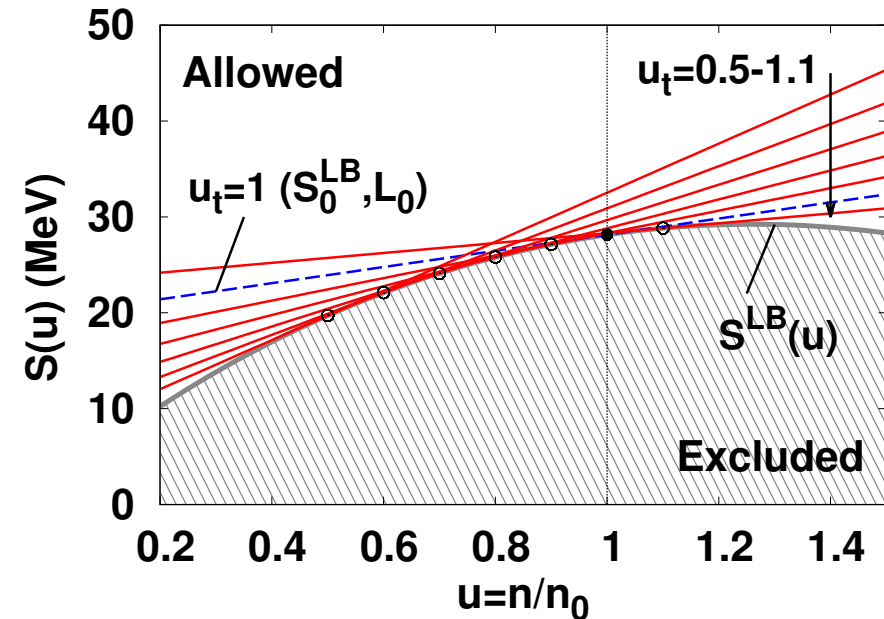
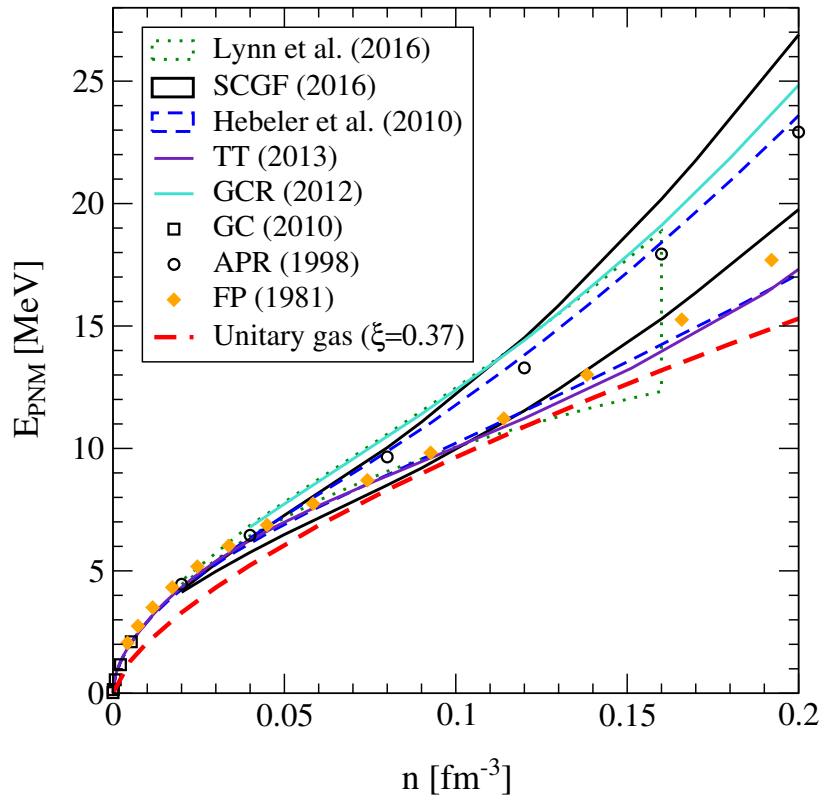
Unitary gas energy seems to be lower bound to neutron-matter energy

- Constraints on S and L

Unitary gas:

- Gas interacting via two-body interactions with infinite scattering length and vanishing effective range
- Then, system has no scale except density, and can be described by a dimensionless parameter, ξ (Bertsch parameter)
- Details of the interaction become irrelevant (universality)
- Experiment and theory: $\xi \approx 0.37$

S and L constraints from lower bound of neutron matter energy



$$S_0 + \frac{L}{3}(u - 1) \geq E_{\text{UG}}^0 u^{2/3} - \left[E_0 + \frac{K_n}{18}(u - 1)^2 \right]$$

$$K_n = K + K_{\text{sym}}$$

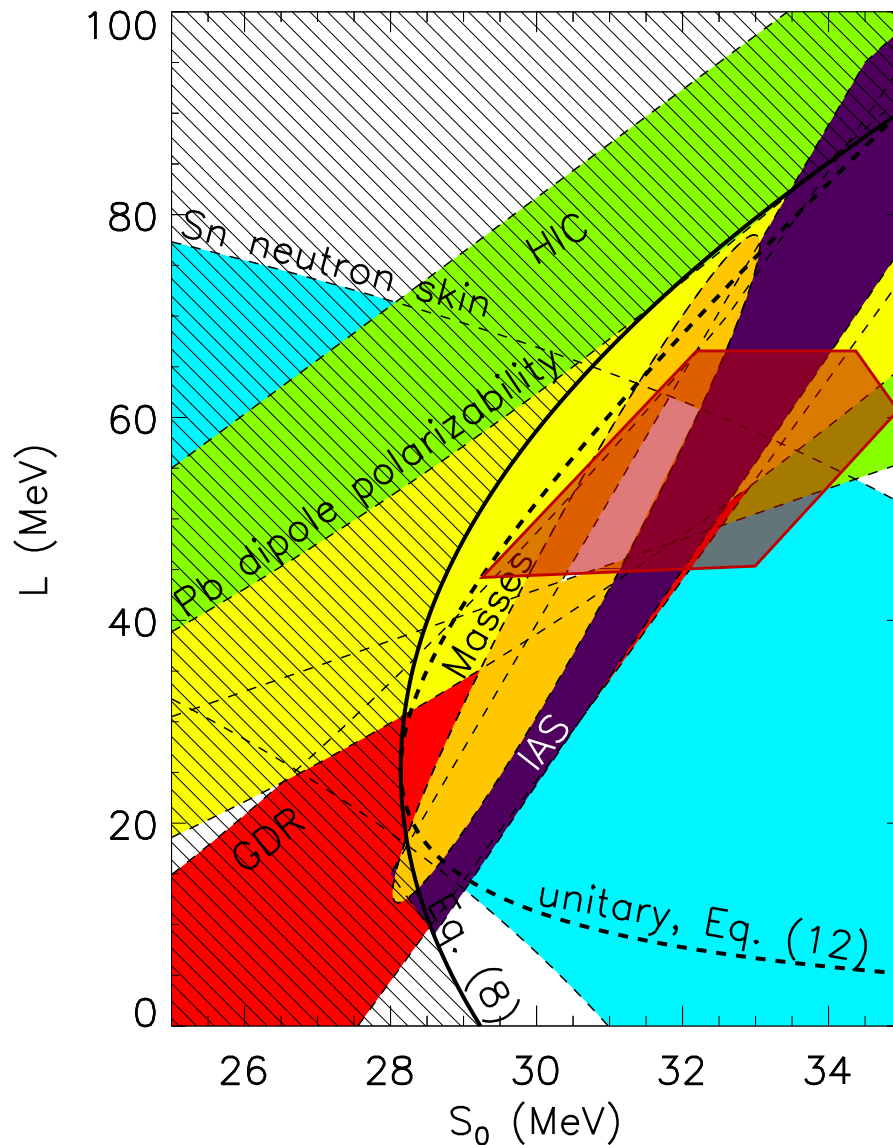
Kolomeitsev, Lattimer, Ohnishi, IT, arXiv:1611.07133

Empirical observation:

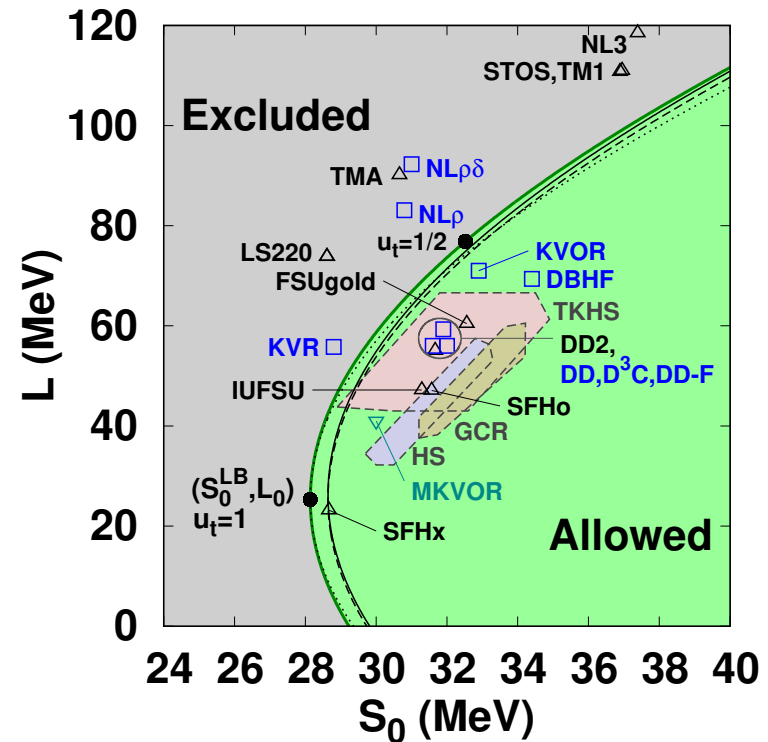
Unitary gas energy seems to be lower bound to neutron-matter energy

➤ Constraints on S and L

S and L constraints from lower bound of neutron matter energy



Kolomeitsev, Lattimer, Ohnishi, IT, arXiv:1611.07133



$$E_0 = -15.5 \text{ MeV}, n_0 = 0.157 \text{ fm}^{-3},$$

$$K = 270 \text{ MeV}, K_{\text{sym}} = 0, \xi = 0.365,$$

$$K_n = K$$

Put constraints on **symmetry energy S** and its density dependence **L** .

$$S_0^{LB} = 28.14 \text{ MeV}, \text{ and } L_0 = 25.28 \text{ MeV}.$$

Summary

- QMC calculations of neutron matter, light nuclei, and n-alpha scattering with local chiral potentials **up to N²LO** including NN and 3N forces can serve as nonperturbative benchmarks.

Gezerlis, IT, Epelbaum, Gandolfi, Hebeler, Nogga, Schwenk, PRL (2013)

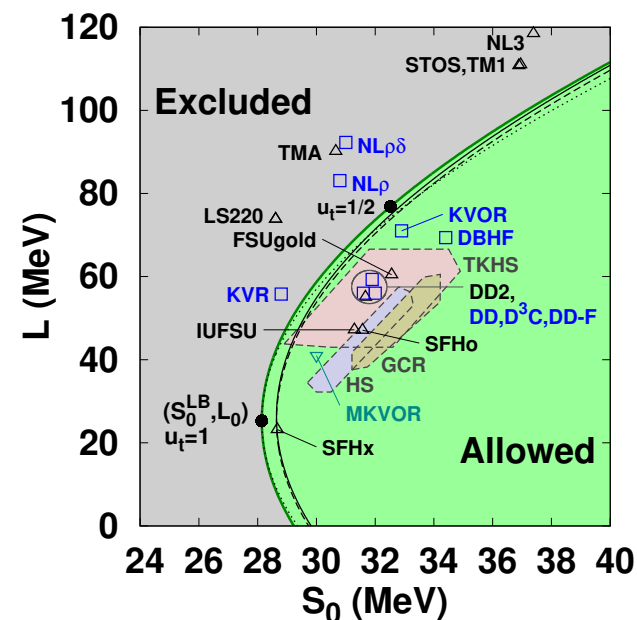
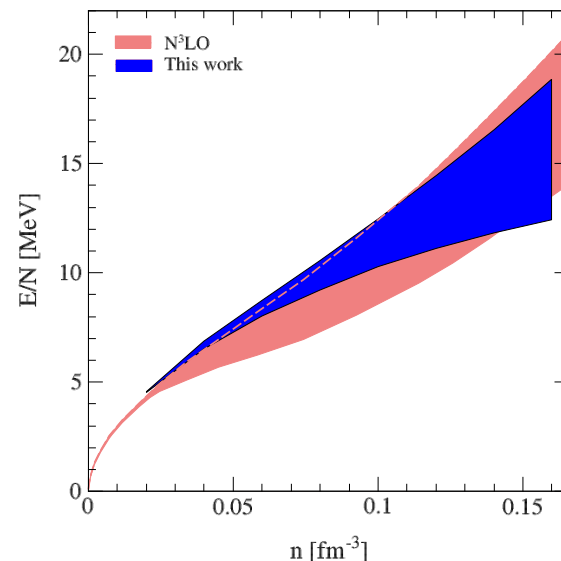
Gezerlis, IT, Epelbaum, Freunek, Gandolfi, Hebeler, Nogga, Schwenk, PRC (2014)

IT, Gandolfi, Gezerlis, Schwenk, PRC (2016)

Lynn, IT, Carlson, Gandolfi, Gezerlis, Schmidt, Schwenk, PRL (2016)

- Chiral interactions at N²LO simultaneously reproduce the properties of $A \leq 5$ systems and of neutron matter, commonly used phenomenological 3N interactions fail.
- Further improvements will allow to determine neutron-matter EOS with improved uncertainties (factor of 2).
- Constraints on symmetry energy and its slope parameter from lower bound of neutron-matter energy.

Kolomeitsev, Lattimer, Ohnishi, IT, arXiv:1611.07133



Thanks

Thanks to my collaborators:

- Technische Universität Darmstadt:
K. Hebeler, J. Lynn, A. Schwenk
- Universität Bochum: E. Epelbaum
- Los Alamos National Laboratory: J. Carlson, S. Gandolfi
- University of Guelph: A. Gezerlis
- Forschungszentrum Jülich: A. Nogga
- Matej Bel University: Evgeni Kolomeitsev
- Stony Brook: Jim Lattimer
- Yukawa Institute Kyoto: Akira Ohnishi



European Research Council
Established by the European Commission



JINA-CEE



Thank you for your attention.